The Effect of Child allowance on Fertility under* a Pay-as-you-go Pension System

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Abstract

This paper presents the economic effects of a child allowance policy using a three-period overlapping-generations model with endogenous fertility. The government operates a pay-as-you-go pension system and provides a child allowance to households. The revenue sources of pension and child allowance are financed by pension premiums and income taxes, respectively, which are both proportional to income. The primary result is that an increase in child allowance decreases per capita income and capital accumulation, whereas the effect on fertility is ambiguous.

Keywords: Endogenous Fertility, Child allowance, Pay-as-you-go pension system, Taxation, OLG model

JEL Classifications: H24, J13, J18

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1 Introduction

Over the last few decades, several developed countries have experienced a declining fertility rate, as shown in Figure 1. This figure illustrates the total fertility rates of the G7 nations from 1961 to 2012. In particular, Japan faces not only lower total fertility but also a rapidly aging society. For a country operating a pay-as-you-go pension system, such as Japan, declining fertility is a serious problem; not only will it slow economic growth but will also lead to the financial collapse of the pension system. During the 1990s, the French government developed a family policy that included a child allowance to increase the total fertility rate; consequently, the fertility rate showed an upward trend, increasing from 1.73% in 1995 to 2.01% in 2013.

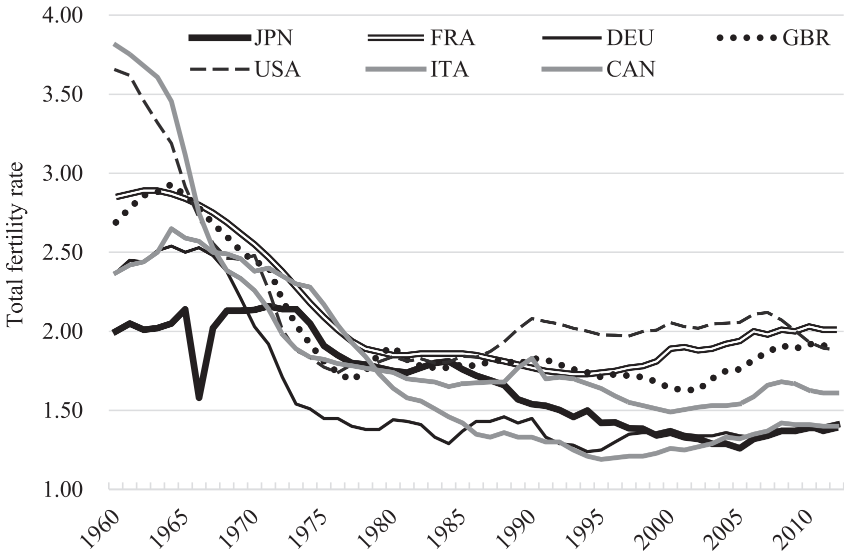

Figure 1. Total fertility rates of G7 nations

Source: World Bank, World Development Indicators 1960-2012.

In this paper, we present the overlapping generations model (OLG)[1] with endogenous fertility, in which a pay-as-you-go pension system is operating, and discuss the effects of child allowance and pension premium rate on macroeconomic factors such as income and capital accumulation. Moreover, we examine whether increasing household child allowance can increase the fertility rate.

There is considerable literature concerning endogenous fertility such as Becker (1996), Becker and Barro (1988), and Barro and Becker (1989). Raut and Srinivasan (1994) and Rupa (1999) analyze the effect of old-age support on capital stock per capita and fertility in an economy where children directly support their aged parents through an income deduction process[2]

With regard to child allowance policies, Galor and Weil (1996) and Fanti and Gori (2009, 2012) present the OLG model of neoclassical growth with endogenous fertility. They demonstrate the possibility of multiple equilibria and analyze the effect of child allowance on fertility and capital accumulation per capita. Galor and Weil (1996) demonstrate the possibility of multiple equilibria using the CES production function, whereas Fanti and Gori (2012) prove that using the Cobb–Douglas production function and considering both the direct and opportunity costs of raising children, they could examine whether fertility increases with an increased child allowance.

This paper aims to study the effect of child allowance on an economy with an old-age support system. We introduce a pay-as-you-go pension system into Fanti and Gori's (2012) model. However, for simplicity, we assume only the monetary aspect of child rearing. Additionally, we consider a labor income tax proportional to income, instead of a lump-sum tax, to finance the child allowance assumed in Fanti and Gori (2012).

1 We introduce endogenous fertility decisions and pension systems into Diamond's (1965) neoclassical growth model.

2 Raut and Srinivasan (1994) assume a Cobb–Douglas production function. Rupa (1999) uses the CES production function.

Within this framework, capital accumulation per capita is shown to be constant in equilibrium. The main result of this paper is that increase in the child allowance and pension premium would decrease per capita income and capital accumulation because these policies slow down the process of capital accumulation in the economy. In addition, although the effect of child allowance on fertility is ambiguous, an increase in the pension premiums could raise the fertility while providing a small child allowance.

The rest of the paper is organized as follows. Section 2 describes the research model, and Section 3 derives the equilibrium. Section 4 discusses the effect of child allowance policy on fertility, and the paper concludes with Section 5.

2 Model

2.1 Household

We consider a three-period OLG economy, where the government operates a pay-as-you-go pension system. Each individual is assumed to live through three periods: childhood, parenthood, and retirement. We suppose that all individuals are endowed with one unit of time for each period and that they do not make decisions during childhood.

In the second period, each individual supplies one unit of labor inelasticity and earns wage w_t. Every individual must simultaneously pay a pension premium qw_t to finance pension benefits. In addition, they must pay labor income tax τw_t, which is spent to provide child allowance. By denoting the pension premium rate and income tax rate as q and τ, respectively, the tax and pension premium burdens are then expressed by qw_t and τw_t, respectively. Moreover, an individual with children receives a child allowance β per child from the government. Then, each individual's disposable income in adulthood is $(1-\tau-q)\, w_t+\beta n_t$, where n_t is the number of children. In addition, under the constraint conditions, each individual decides how much to consume c_t, save s_t, and decides the number of children n_t. However, the rearing cost is m for each child[3]. Taking c as the numeraire, we assume the price is one, and the household's budget constraint in parenthood is

$$c_t + s_t + (m - \beta)\, n_t = (1 - \tau - q)\, w_t, \;\; \tau, q \in (0,1).$$

In the third period, each individual retires and consumes. The consumption value is calculated as the sum of private savings, interest, and pension benefit:

$$c_{t+1} = (1 + r_{t+1})\, s_t + b_{t+1},$$

where r_{t+1} and b_{t+1} represent the interest rate and pension benefit, which each individual receives in the retirement period, respectively. Combining the above equations, we obtain the following lifetime budget constraint:

$$c_t + \frac{c_{t+1}}{1 + r_{t+1}} + (m - \beta)\, n_t - \frac{b_{t+1}}{1 + r_{t+1}} = (1 - \tau - q)\, w_t \tag{1}$$

Each individual derives utility from consumption in the parenthood and retirement periods, and the number of children. Then, we specify the utility function of each individual born at time $t - 1$ as follows:

$$u_{t-1} = (1 - \phi)\log c_t + \gamma \log c_{t+1} + \phi \log n_t, \;\; \phi \in (0,1) \tag{2}$$

where $(1 - \phi)$ and ϕ denote preference toward consumption in parenthood and the number of children, respectively, and γ denotes the discount factor.

By maximizing the above utility function subject to equation (1), we get the following first-order conditions:

$$\frac{c_{t+1}}{c_t} \cdot \frac{1 - \phi}{\gamma} = 1 + r_{t+1} \tag{i}$$

$$\frac{c_t}{n_t} \cdot \frac{\phi}{1 - \phi} = m - \beta. \tag{ii}$$

Substituting equations (i) and (ii) into equation (1), the optimal allocations are determined as

3 Borldrin and Jones (2002) consider the amount of rearing cost m and opportunity cost $n_t w_t$. For simplicity, we assume that only the rearing cost m per child is required.

$$c_t = \frac{1-\phi}{1+\gamma}\left[(1-\tau-q)\,w_t + \frac{b_{t+1}}{1+r_{t+1}}\right], \tag{3}$$

$$c_{t+1} = \frac{\gamma}{1+\gamma}\,[(1+r_{t+1})\,(1-\tau-q)\,w_t + b_{t+1}], \tag{4}$$

$$n_t = \frac{\phi}{(1+\gamma)\,(m-\beta)}\left[(1-\tau-q)\,w_t + \frac{b_{t+1}}{1+r_{t+1}}\right]. \tag{5}$$

From equation (4), optimal savings is obtained as follows:

$$s_t = \frac{1}{1+\gamma}\left[\gamma\,(1-\tau-q)\,w_t - \frac{b_{t+1}}{1+r_{t+1}}\right]. \tag{6}$$

2.2 Firm

The representative firm acts in a perfectly competitive market. There is a single consumption good produced using the constant returns to scale production function, employing capital and labor. We assume the production function is $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where Y_t, K_t, and $L_t = N_t$ represent output, capital, and labor input in period t, respectively; A > 0 is a scale parameter; and $\alpha \in (0,1)$ is the weight of capital in the technology. Then, the intensive production function is written as

$$y_t = Ak_t^{\alpha}, \tag{7}$$

where $y_t \equiv \frac{Y_t}{N_t}$ and $k_t \equiv \frac{Y_t}{N_t}$ are output and capital per capita, respectively. We assume that capital fully depreciates at the end of every period and that the price of the consumption good is one.

Solving for profit maximization under the interest rate r_t and the wage rate w_t, we get the following marginal conditions:

$$1 + r_t = \alpha A k_t^{\alpha-1}, \tag{8}$$

$$w_t = (1-\alpha)\,Ak_t^{\alpha}. \tag{9}$$

2.3 Government

The government operates a pay-as-you-go pension system and child allowance policy. In every period, the government imposes a pension premium and labor income tax on workers. In period t, the size of the working population is expressed by N_t. We define the growth factor of the young generation in period t as $N_t = n_{t-1}N_{t-1}$.

The contribution revenues and payments of pension system at time t are denoted by qw_tN_t and b_tN_{t-1}, respectively.

The tax revenue in period t that finances child allowance is denoted by τw_tN_t and the expenditures concerning child allowance are expressed by βn_tN_t. Therefore, a balanced government budget in period t must satisfy the following equations:

$$b_t = qw_tn_{t-1}, \tag{10}$$

$$\tau w_t = \beta n_t. \tag{11}$$

3 Equilibrium

In this section, we consider the dynamics of capital stock and fertility in the equilibrium. Substituting equations (10) and (11) into equation (5), the number of children in period t is

$$n_t = \frac{\phi(1+r_{t+1})(1-q)w_t}{(1+r_{t+1})(1+\gamma)(m-\beta)+\phi[\beta(1+r_{t+1})-qw_{t+1}]}. \tag{12}$$

In addition, substituting the above equation into equation (6), individual savings is represented as

$$s_t = (1-q)w_t\frac{\gamma(1+r_{t+1})(m-\beta)-\phi qw_{t+1}}{(1+r_{t+1})(1+\gamma)(m-\beta)+\phi[\beta(1+r_{t+1})-qw_{t+1}]}. \tag{13}$$

The dynamics of capital stock is represented by

$$K_{t+1} = s_tN_t.$$

Dividing the above equation by N_t, the dynamics of capital stock is described by the following equation:

$$k_{t+1} = \frac{s_t}{n_t}. \tag{14}$$

Combining equations (8), (9), (12), (13), and (14), we get

$$k_{t+1} = k^{*}(\beta, q) = \frac{\alpha\gamma(m-\beta)}{\phi(\alpha + q(1-\alpha))}, \tag{15}$$

where we assume $m > \beta$[4].

In this economy, the capital stock per capita is constant in the long run, as shown by Figure 2; therefore, fertility is also constant.

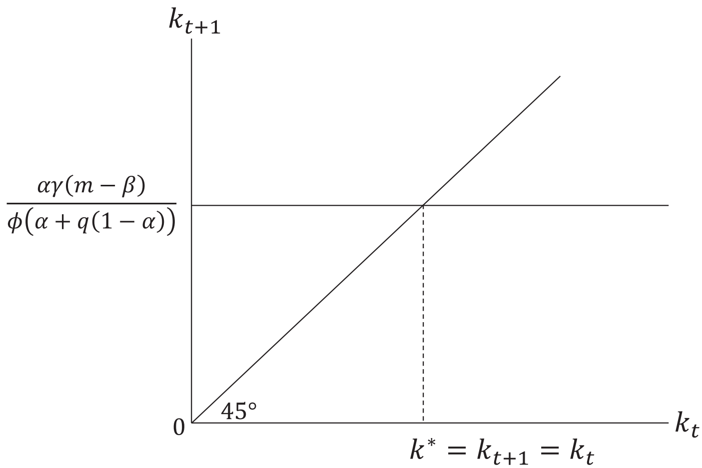

Figure 2. Per capita capital accumulation in the economy

4 Policy Analysis

This section presents the effects of child allowance and contribution rate on fertility and per capita income.

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4 Considering both the direct and opportunity costs of raising children, we get the dynamics similar to Fanti and Gori (2012). Then, without the condition of $m > \beta$, we can discuss the dynamics in the economy.

Differentiating equation (15) with respect to β and q, each partial derivative is

$$\frac{\partial k^*(\beta, q)}{\partial \beta} = -\frac{\alpha\gamma}{\phi(\alpha + q(1-\alpha))} < 0, \tag{iii}$$

$$\frac{\partial k^*(\beta, q)}{\partial q} = -\frac{\alpha\gamma(1-\alpha)(m-\beta)}{\phi(\alpha + q(1-\alpha))^2} < 0, \tag{iv}$$

That is, an increase in child allowance and contribution rate decreases capital accumulation in the economy, as described by Figures 3 and 4. Therefore, the following proposition holds:

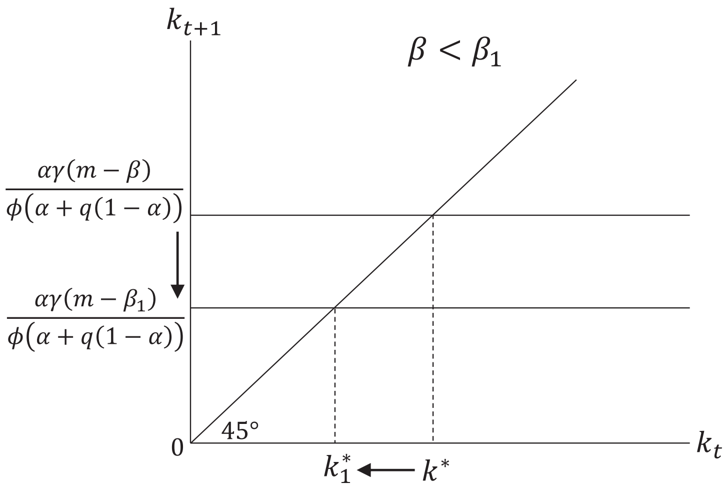

Figure 3. Effect of an increase in β

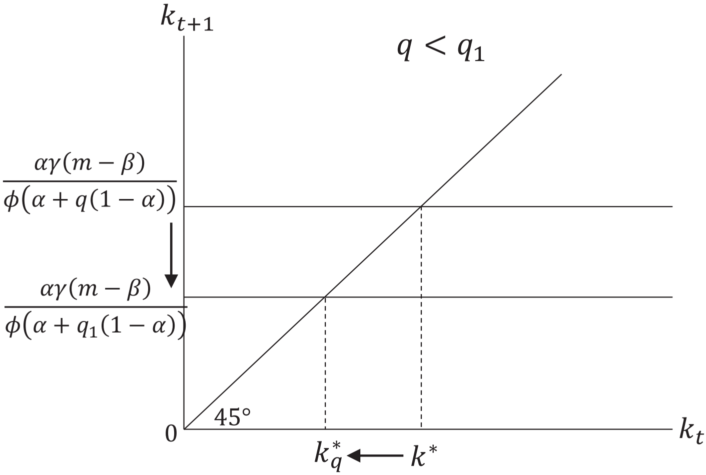

Figure 4. Effect of an increase in *q*

Proposition 1. Per capita income increases monotonically with child allowance β and pension contribution rate q.

Proof. Substituting equation (15) into equation (7), the production function per capita is expressed by

$$y^* = A\,(k^*)^{\alpha} = A\left(\frac{\alpha\gamma\,(m-\beta)}{\phi\,(\alpha+q\,(1-\alpha))}\right)^{\alpha}.$$

For any $\beta > 0$ and $q > 0$,

$$\frac{\partial y^*}{\partial \beta} = (-1)\,\alpha\,A\,(m-\beta)^{\alpha-1}\left(\frac{\alpha\gamma}{\phi\,(\alpha+q\,(1-\alpha))}\right)^{\alpha} < 0,$$

$$\frac{\partial y^*}{\partial q} = (-\alpha)\,\frac{(1-\alpha)}{(\alpha+q\,(1-\alpha))}\,A\left(\frac{\alpha\gamma\,(m-\beta)}{\phi\,(\alpha+q\,(1-\alpha))}\right)^{\alpha} < 0.$$

■

To analyze the effect of child allowance on the fertility rate, we rewrite the demand for children in the equilibrium:

$$n^* = \frac{\phi(1+r^*)(1-q)w^*}{(1+r^*)(1+\gamma)(m-\beta)+\phi[\beta(1+r^*)-qw^*]}. \tag{16}$$

To ensure that the fertility rate is positive, we suppose $\beta(1+r^*) > qw^*$.

We now analyze the effect of child allowance and contribution rate on fertility. Differentiating equation (16) with respect to β and w^*, we have

$$\frac{\partial n^*}{\partial \beta} = \frac{\phi(1+r^*)^2(1-q)w^*(1-\phi+\gamma)}{[(1+r^*)(1+\gamma)(m-\beta)+\phi[\beta(1+r^*)-qw^*]]^2}, \tag{v}$$

$$\frac{\partial n^*}{\partial w^*} = \frac{\phi(1+r^*)^2(1-q)[(1+\gamma)(m-\beta)+\phi\beta]}{[(1+r^*)(1+\gamma)(m-\beta)+\phi[\beta(1+r^*)-qw^*]]^2} > 0. \tag{vi}$$

However, since an increase in β has both direct and indirect impacts, we consider the total effect of child allowance on fertility. Differentiating equation (16) with respect to β the total derivative is expressed as

$$\frac{\partial n^*}{\partial \beta} = \frac{\partial n^*}{\partial \beta} + \frac{\partial n^*}{\partial w^*}\cdot\frac{\partial w^*}{\partial k^*}\cdot\frac{\partial k^*}{\partial \beta}. \tag{17}$$

The first term of RHS of equation (17) represents the direct impact of β on fertility. From equations (iii), (v), and (vi),

$$\frac{\partial n^*}{\partial \beta} = \underset{+}{\frac{\partial n^*}{\partial \beta}} + \underset{+}{\frac{\partial n^*}{\partial w^*}}\cdot\underset{+}{\frac{\partial w^*}{\partial k^*}}\cdot\underset{-}{\frac{\partial k^*}{\partial \beta}},$$

therefore, the sign of $\frac{\partial n^*}{\partial \beta}$ is ambiguous.

In addition, we examine the effect of contribution rate on fertility. Differentiating equation (16) with respect to q, we have

$$\frac{\partial n^*}{\partial q^*} = -\frac{\phi(1+r^*)w^*[(1+r^*)(1+\gamma)(m-\beta)+\phi[\beta(1+r^*)-w^*]]}{(1+r^*)(1+\gamma)(m-\beta)+\phi[\beta(1+r^*)-qw^*]}. \tag{vii}$$

From the above equation, the effect of pension premium on fertility is also ambiguous. In an economy where child allowance is sufficiently large (since the sign of $\frac{\partial n^*}{\partial q}$ is negative), a decrease in pension premium could increase fertility.

5 Conclusions

We present the effects of child allowance and pension premium on macroeconomic factors, such as per capita income, in the OLG model with endogenous fertility. Moreover, the government operates a pay-as-you-go pension system and provides a child allowance. The results indicate that increase in the child allowance and pension premium have slowing effects on economic growth. However, the effect of child allowance on fertility is not clear. With regard to the pension premium, a decrease could have a positive effect on fertility if the child allowance is sufficiently large because of the trade-off between child allowance and pension payment.

To provide generous social-welfare systems, governments must impose higher taxes on households to cover the necessary expenses. That is, households cannot receive benefits without any additional cost in the form of tax and pension premiums. Additionally, higher taxes reduce the disposable income and savings of households. As a result, capital stock declines and economic growth slows. Financial support alone is not enough to raise fertility and sustain economic growth. For example, in France, where the total fertility rate has maintained an increasing trend since 1995, institutions are established that support families with children, including a well-developed maternity and parental leave system. It is important to systematically support childcare in this model.

Finally, it would be interesting to extend this model with a well-developed maternity and parental leave system, and analyze the policy effect on fertility.

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